Quasi-Metric Relativity

by

Dag Østvang

Institutt for Fysikk, Norges Teknisk-Naturvitenskapelige Universitet, NTNU
N-7491 Trondheim, Norway

Abstract

This is a survey of a new type of relativistic space-time framework; the so-called quasi-metric framework. The basic geometric structure underlying quasi-metric relativity is a one-parameter family $\mathbf{g}_t$ of Lorentzian 4-metrics parametrized by a global time function t. A linear and symmetric affine connection $\overset{\star}{\nabla}$ compatible with the family $\mathbf{g}_t$ is defined, giving rise to equations of motion.

Furthermore a quasi-metric theory of gravity, including field equations and local conservation laws, is presented. The field equations have only one dynamical degree of freedom coupled explicitly to matter, but there is also a second, implicit dynamical degree of freedom. The existence of this implicit dynamical degree of freedom makes the field equations unsuitable for a standard PPN-analysis. This implies that the experimental status of the theory is not completely clear at this point in time. The theory seems to be consistent with a number of cosmological observations and it satisfies all the classical solar system tests however. Moreover, in its non-metric sector the new theory has experimental support where General Relativity fails or is irrelevant.

1 Introduction

Interest in alternative classical theories of gravity has mainly focused on the class of metric theories, defined by the postulates [1]

- Space-time is equipped with a single Lorentzian metric field $\mathbf{g}$,
- The world lines of inertial test particles are geodesics of $\mathbf{g}$,
- In the local Lorentz frames, the non-gravitational physics is as in Special Relativity.

One reason for the neglect of non-metric theories is probably the successes of the leading metric theory, General Relativity (GR): constructing alternative theories not deviating too significantly in structure from GR seems compelling if one is not prepared to risk immediate conflict with observation.

But another reason is possibly the belief that theories which do not satisfy the above postulates necessarily fail to satisfy the Einstein Equivalence Principle (EEP) defined by the restrictions [1]

- The trajectories of uncharged test particles do not depend on their internal composition (this is the Weak Equivalence Principle),
- The outcomes of local non-gravitational test experiments do not depend on the velocity of the apparatus (this is called local Lorentz invariance),
- The outcomes of local non-gravitational test experiments do not depend on when or where they are performed (this is called local positional invariance).

Since the empirical evidence supporting the EEP seems formidable [1], constructing a theory violating it probably would be a waste of time. But is it really true that theories not satisfying all the said postulates necessarily violate the EEP?

No, it is not. It can be shown that it is possible to construct a type of relativistic space-time framework not satisfying the first two postulates but where the EEP still holds [2]. This framework defines the geometrical basis for Quasi-Metric Relativity (QMR), just as semi-Riemannian geometry defines the geometrical basis for metric relativity.

A general physical motivation for introducing the quasi-metric framework is found directly in the particular global structure of quasi-metric space-time. That is, the geometric structure behind QMR is constructed to yield maximal predictive power with regard to the large-scale properties of space-time. The basic idea that makes this possible is that since the Universe is unique, so should the nature of its global evolution be. That is, there should be no reason to treat the Universe as a purely gravitationally dynamic system and its global evolution should not depend on any particular choice of initial conditions. This means that the global evolution of the Universe should be explicitly included into the geometric structure of quasi-metric space-time as some sort of prior geometric property. It is natural to call the global evolution of the Universe "non-kinematical" since by construction, this evolution is unaffected by dynamics. In other words, in QMR the global evolution of the Universe is described as a non-kinematical cosmic expansion. One important consequence of this is that a global arrow of time exists as an intrinsic, geometrical property of quasi-metric space-time. The quasi-metric framework thus represents an attractive solution of the problem of time-asymmetry. (See e.g. [3] and references therein for more on this problem.)

The quasi-metric framework and some of its predictions will be described in some detail in the following. Note that since this paper is intended to be a not too lengthy

introduction to QMR, derivations of formulae are in general omitted. However, more detailed derivations can often be found in [2].

2 The quasi-metric space-time framework

2.1 Basic mathematical structure

As mentioned in the introduction, the basic premise of the quasi-metric framework is that the cosmic expansion should be described as a non-kinematical phenomenon. To fulfill this premise it is necessary that the canonical description of space-time is taken as fundamental. Furthermore, to ensure that the cosmic expansion really is global and directly integrated into the geometric structure of quasi-metric space-time, it is necessary to introduce *the global time function* t representing an extra, degenerate time dimension (see below). This extra time dimension must be degenerate since it is designed to describe the cosmic expansion as a non-kinematical global scale change between gravitational and non-gravitational systems. We will elaborate on this in the following where we define the quasi-metric framework precisely in terms of geometrical structures on a differentiable manifold.

Mathematically the quasi-metric framework can be described by first considering a 5-dimensional product manifold $\mathcal{M}\times\mathbf{R}_1$, where $\mathcal{M} = \mathcal{S}\times\mathbf{R}_2$ is a (globally hyperbolic) Lorentzian space-time manifold, $\mathbf{R}_1$ and $\mathbf{R}_2$ are two copies of the real line and $\mathcal{S}$ is a compact Riemannian 3-dimensional manifold (without boundaries). We see that it is natural to interpret t as a coordinate on $\mathbf{R}_1$. Besides, the product topology of $\mathcal{M}$ implies that once t is given, there must exist a "preferred" ordinary time coordinate x^0 on $\mathbf{R}_2$ such that x^0 scales like ct. It is very convenient to choose a time coordinate x^0 which scales like ct since this means that x^0 is in some sense a mirror of t and that they thus are "equivalent" global time coordinates. A coordinate system with a global time coordinate of this type we call *a global time coordinate system* (GTCS). Hence, expressed in a GTCS $\{x^\mu\}$ (where μ can take any value $0-3$), x^0 is interpreted as a global time coordinate on $\mathbf{R}_2$ and $\{x^j\}$ (where j can take any value $1-3$) as spatial coordinates on $\mathcal{S}$. The class of GTCSs is a set of preferred coordinate systems inasmuch as the equations of QMR take special forms in a GTCS.

We now equip $\mathcal{M}\times\mathbf{R}_1$ with two degenerate 5-dimensional metrics $\bar{\mathbf{g}}_t$ and $\mathbf{g}_t$. By definition the metric $\bar{\mathbf{g}}_t$ represents a solution of field equations, and from $\bar{\mathbf{g}}_t$ one can construct the "physical" metric $\mathbf{g}_t$ which is used when comparing predictions to experiments. To reduce the 5-dimensional space-time $\mathcal{M}\times\mathbf{R}_1$ to a 4-dimensional space-time we just slice

the 4-dimensional sub-manifold $\mathcal{N}$ determined by the equation $x^0 = ct$ (using a GTCS) out of $\mathcal{M}\times\mathbf{R}_1$. Moreover, in $\mathcal{N}$ $\bar{\mathbf{g}}_t$ and $\mathbf{g}_t$ are interpreted as one-parameter metric families. Thus by construction $\mathcal{N}$ is a 4-dimensional space-time manifold equipped with two one-parameter families of Lorentzian 4-metrics parameterized by the global time function t. This is the general form of the quasi-metric space-time framework. We will call $\mathcal{N}$ *a quasi-metric space-time manifold.* And the reason why $\mathcal{N}$ is different from a Lorentzian space-time manifold is that the affine connection compatible with any metric family is non-metric. This means that, while it is always possible to equip $\mathcal{N}$ with the single metric obtained by inserting the explicit substitution $t = x^0/c$ into $\mathbf{g}_t$, this metric is useless for other purposes than taking scalar products. That is, since the affine structure on $\mathcal{N}$ is inherited from the affine structure on $\mathcal{M}\times\mathbf{R}_1$, and since that affine structure is not compatible with any single metric on $\mathcal{N}$ (see below), one *must* separate between ct and x^0 in $\mathbf{g}_t$.

From the definition of quasi-metric space-time we see that it is constructed as consisting of two mutually orthogonal foliations: on the one hand space-time can be sliced up globally into a family of 3-dimensional space-like hypersurfaces (called the fundamental hypersurfaces (FHSs)) by the global time function t, on the other hand space-time can be foliated into a family of time-like curves everywhere orthogonal to the FHSs. These curves represent the world lines of a family of hypothetical observers called the fundamental observers (FOs), and the FHSs together with t represent a preferred notion of space and time. That is, the equations of any theory of gravity based on quasi-metric geometry should depend on quantities obtained from this preferred way of splitting up space-time into space and time. But notice that the structure of quasi-metric space-time has no effects on local non-gravitational test experiments.

Next we describe the affine structure on $\mathcal{N}$. (Note that we introduce the coordinate notation $g_{(t)\mu\nu}$ where the parenthesis is put in to emphasize that these are the components of a one-parameter family of metrics rather than those of a single metric.) Again we start with the corresponding structure on $\mathcal{M}\times\mathbf{R}_1$. To find that we should think of the metric family $\mathbf{g}_t$ as one single degenerate metric on $\mathcal{M}\times\mathbf{R}_1$, where the degeneracy manifests itself via the condition $\mathbf{g}_t(\frac{\partial}{\partial t}, \cdot)\equiv 0$. The natural way to proceed is to determine a torsion-free, metric-compatible 5-dimensional "degenerate" connection $\overset{\star}{\nabla}$ on $\mathcal{M}\times\mathbf{R}_1$ from the metric-preserving condition

$$\frac{\partial}{\partial t}\mathbf{g}_t(\mathbf{y}_t, \mathbf{z}_t) = \mathbf{g}_t(\overset{\star}{\nabla}_{\frac{\partial}{\partial t}}\mathbf{y}_t, \mathbf{z}_t) + \mathbf{g}_t(\mathbf{y}_t, \overset{\star}{\nabla}_{\frac{\partial}{\partial t}}\mathbf{z}_t), \tag{1}$$

involving arbitrary families of vector fields $\mathbf{y}_t$ and $\mathbf{z}_t$ in $\mathcal{M}$. It turns out that it is possible to find a unique candidate connection satisfying equation (1) in general and differing from

the usual Levi-Civita connection only via connection coefficients containing t. This candidate connection is determined from the in general non-zero connection coefficients $\overset{\star}{\Gamma}{}^{\alpha}_{\mu t}$ which must be equal to $\frac{1}{2}g^{\alpha\sigma}_{(t)}\frac{\partial}{\partial t}g_{(t)\sigma\mu}$ (we use Einstein's summation convention throughout), since other connection coefficients containing t must vanish identically.

But the above-mentioned candidate degenerate connection has one undesirable property, namely that it does not in general ensure that the unit normal vector field family $\mathbf{n}_t$ of the FHSs (with the property $\mathbf{g}_t(\mathbf{n}_t, \mathbf{n}_t) = -1$) is parallel-transported along $\frac{\partial}{\partial t}$. It would be natural to require this property, i.e., $\overset{\star}{\nabla}$ should guarantee that

$$\overset{\star}{\nabla}_{\frac{\partial}{\partial t}}\mathbf{n}_t = 0, \tag{2}$$

since if equation (2) does not hold the resulting equations of motion will not be identical to the geodesic equation derived from $\overset{\star}{\nabla}$. However, we may try to construct a unique degenerate connection which satisfies equation (2) and is identical to the above-mentioned candidate connection for those particular cases when the candidate connection satisfies equation (2). This is possible since the dependence of $\mathbf{g}_t$ on t cannot be arbitrary. That is, the dependence of $\mathbf{g}_t$ on t can be inferred independently and takes a particular form (see equation (12) below), making it possible to construct a unique degenerate connection which satisfies both equations (1) and (2) (given the particular dependence of $\mathbf{g}_t$ on t). This unique connection is determined from the form the connection coefficients take in a GTCS, and the result is

$$\overset{\star}{\Gamma}{}^{\alpha}_{\mu t} = \frac{1}{t}\delta^{\alpha}_{i}\delta^{i}_{\mu}, \qquad \overset{\star}{\Gamma}{}^{\alpha}_{t\mu} \equiv \overset{\star}{\Gamma}{}^{\alpha}_{\mu t}, \qquad \overset{\star}{\Gamma}{}^{\alpha}_{\nu\mu} \equiv \Gamma^{\alpha}_{(t)\nu\mu}, \tag{3}$$

where $\Gamma^{\alpha}_{(t)\nu\mu}$ are the connection coefficients of the family ∇_t of Levi-Civita connections defined from the collection of single Lorentzian metrics on $\mathcal{M}$. The restriction of $\overset{\star}{\nabla}$ to $\mathcal{N}$ is trivial since it does not involve any projections. (That is, to apply $\overset{\star}{\nabla}$ in $\mathcal{N}$ one just applies it in the sub-manifold $x^0 = ct$ in a GTCS.) Notice that other degenerate connection coefficients than those given in equation (3) vanish identically. This implies that the gradient of the global time function is covariantly constant, i.e., $\overset{\star}{\nabla}_{\frac{\partial}{\partial t}} dt = \overset{\star}{\nabla}_{\frac{\partial}{\partial x^\mu}} dt = 0$.

It is in general possible to write $\mathbf{g}_t$ as a sum of two terms

$$\mathbf{g}_t = -\mathbf{g}_t(\mathbf{n}_t, \cdot)\otimes\mathbf{g}_t(\mathbf{n}_t, \cdot) + \mathbf{h}_t, \tag{4}$$

where $\mathbf{h}_t$ is the family of spatial metrics intrinsic to the FHSs. Then equations (1) and (2) imply that

$$\overset{\star}{\nabla}_{\frac{\partial}{\partial t}}\mathbf{g}_t = 0, \qquad \overset{\star}{\nabla}_{\frac{\partial}{\partial t}}\mathbf{h}_t = 0, \tag{5}$$

thus the degenerate connection is compatible with the metric family as asserted.

2.2 General equations of motion

Now we want to use the above defined affine structure on $\mathcal{N}$ to find equations of motion for test particles in $(\mathcal{N}, \mathbf{g}_t)$. Let λ be an affine parameter along the world line in $\mathcal{N}$ of an arbitrary test particle. (In addition to the affine parameter λ, t is also a (non-affine) parameter along any non-space-like curve in $\mathcal{N}$.) Using an arbitrary coordinate system (not necessarily a GTCS) we may define coordinate vector fields $\frac{\partial}{\partial x^\alpha}$; then $\frac{dt}{d\lambda}\frac{\partial}{\partial t} + \frac{dx^\alpha}{d\lambda}\frac{\partial}{\partial x^\alpha}$ is the coordinate representation of the tangent vector field $\frac{\partial}{\partial \lambda}$ along the curve. We then define the degenerate covariant derivative along the curve as

$$\overset{\star}{\nabla}_{\frac{\partial}{\partial\lambda}} \equiv \frac{dt}{d\lambda}\overset{\star}{\nabla}_{\frac{\partial}{\partial t}} + \frac{dx^\alpha}{d\lambda}\overset{\star}{\nabla}_{\frac{\partial}{\partial x^\alpha}}. \tag{6}$$

A particularly important family of vector fields is the 4-velocity tangent vector field family $\mathbf{u}_t$ along a curve. By definition we have

$$\mathbf{u}_t \equiv u^\alpha_{(t)}\frac{\partial}{\partial x^\alpha} \equiv \frac{dx^\alpha}{d\tau_t}\frac{\partial}{\partial x^\alpha}, \tag{7}$$

where τ_t is the proper time as measured along the curve.

The equations of motion are found by calculating the covariant derivative of 4-velocity tangent vectors along themselves using the connection in $(\mathcal{N}, \mathbf{g}_t)$. According to the above, this is equivalent to calculating $\overset{\star}{\nabla}\mathbf{u}_t$ along $\frac{\partial}{\partial \tau_t}$. Using the coordinate representation of $\frac{\partial}{\partial \tau_t}$ we may thus define the vector field $\overset{\star}{\mathbf{a}}$ by

$$\overset{\star}{\mathbf{a}} \equiv \overset{\star}{\nabla}_{\frac{\partial}{\partial \tau_t}}\mathbf{u}_t = \Big(\frac{dt}{d\tau_t}\overset{\star}{\nabla}_{\frac{\partial}{\partial t}} + \frac{dx^\alpha}{d\tau_t}\overset{\star}{\nabla}_{\frac{\partial}{\partial x^\alpha}}\Big)\mathbf{u}_t \equiv \frac{dt}{d\tau_t}\overset{\star}{\nabla}_{\frac{\partial}{\partial t}}\mathbf{u}_t + \mathbf{a}_t. \tag{8}$$

We call this vector field the "degenerate" 4-acceleration. We need to have an independent expression for the degenerate acceleration field $\overset{\star}{\mathbf{a}}$. This can be found by calculating the extra term $\frac{dt}{d\tau_t}\overset{\star}{\nabla}_{\frac{\partial}{\partial t}}\mathbf{u}_t$ at the right hand side of equation (8). To do that it is convenient to introduce the 3-velocity $\mathbf{w}_t$ of an arbitrary test particle as seen by the FOs. That is, one may split up the tangent 4-velocity into parts respectively normal and tangential to the FHSs, i.e.

$$\mathbf{u}_t = \overset{\star}{\gamma}(c\mathbf{n}_t + \mathbf{w}_t), \qquad \overset{\star}{\gamma} \equiv (1 - \frac{w^2}{c^2})^{-1/2} = \frac{d\tau_{\mathcal{F}}}{d\tau_t}, \tag{9}$$

where w^2 is the square of $\mathbf{w}_t$ and $d\tau_{\mathcal{F}} \equiv N dt$ is the proper time interval of the local FO. Here N is the lapse function field (as expressed in a GTCS) of the FOs. Note that $\mathbf{w}_t$ is an object intrinsic to the FHSs since $\mathbf{g}_t(\mathbf{n}_t, \mathbf{w}_t) \equiv 0$. Moreover, from the general dependence of $\bar{\mathbf{g}}_t$ and $\mathbf{g}_t$ on t given in equation (12) below, together with the requirement that w^2

should be independent of t, we can find the coordinate expressions in a GTCS of $\mathbf{n}_t$ and $\mathbf{w}_t$. These are given by [2]

$$n^0_{(t)} = N^{-1}, \qquad n^j_{(t)} = -\frac{t_0}{t}\frac{N^j}{N}, \qquad w^0_{(t)} = 0, \qquad w^j_{(t)} = \frac{dx^j}{d\tau_{\mathcal{F}}} + \frac{t_0}{t}\frac{N^j}{N}c, \tag{10}$$

where the $N^j_{(t)} \equiv \frac{t_0}{t} N^j$ are the components in a GTCS of the shift vector field family of the FOs. Here t_0 is just some arbitrary epoch setting the scale of the spatial coordinates. Note that N and N^j do not depend explicitly on t. Also note that the proper time interval $d\tau_{\mathcal{F}} \equiv N dt$ may in principle be integrated along the world line of any FO given the implicit dependence $N(x^\mu(t))$ in $(\mathcal{N}, \mathbf{g}_t)$. This means that there is a direct relationship between t and the proper time elapsed for any FO. So, since N is non-negative by definition, t must be increasing in the forward direction of time for any FO.

By using equations (3), (9) and (10) we may now show that $\stackrel{\star}{\nabla}_{\frac{\partial}{\partial t}} \mathbf{u}_t$ vanishes, so we in fact have $\stackrel{\star}{\mathbf{a}} = \mathbf{a}_t$ from equation (8). The coordinate expression for $\stackrel{\star}{\mathbf{a}}$ then yields equations of motion, namely

$$\frac{d^2 x^\alpha}{d\lambda^2} + \Big(\stackrel{\star}{\Gamma}{}^\alpha_{t\sigma} \frac{dt}{d\lambda} + \stackrel{\star}{\Gamma}{}^\alpha_{\beta\sigma} \frac{dx^\beta}{d\lambda} \Big) \frac{dx^\sigma}{d\lambda} = \Big(\frac{d\tau_t}{d\lambda} \Big)^2 a^\alpha_{(t)}. \tag{11}$$

Equation (11) is the geodesic equation obtained from $\stackrel{\star}{\nabla}$ and this implies that inertial test particles follow geodesics of $\stackrel{\star}{\nabla}$. Note that while the form of equation (11) is valid in general coordinates, the form of $\stackrel{\star}{\Gamma}{}^\alpha_{\mu t}$ given in equation (3) is not.

To get the correspondence with metric gravity we formally set $\frac{t_0}{t} = 1$ and then take the limit $t \to \infty$ in equations (3), (10) and (11). The equations of motion (11) then reduce to the usual geodesic equation in metric gravity. This limit represents the so-called *metric approximation* where the metric family $\mathbf{g}_t$ does not depend on t. That is, in the metric approximation $\mathbf{g}_t$ can be identified with one single Lorentzian metric $\mathbf{g}$. Notice that in QMR, metric approximations are meaningful for isolated systems only. This is why correspondences between QMR and metric gravity can be found in e.g., the solar system but not for cosmology (see section 4).

However, except for the metric approximation, $\mathbf{g}_t$ should not be identified with any single Lorentzian metric and equation (11) does not reduce to the usual geodesic equation in metric gravity due to terms explicitly depending on t. That is, in QMR inertial test particles do not move as if they were following geodesics of any single space-time metric. Also note that the equations of motion (11) do not violate local Lorentz invariance. To see this, observe that the connection coefficients may be made to vanish in any local inertial frame so that equation (11) takes its special relativistic form.

3 Quasi-metric gravity

3.1 Basic principles

At this point two questions naturally arise, namely

- What is the role of t in the metric families $\mathbf{g}_t$ and $\bar{\mathbf{g}}_t$? and
- Why should it be preferable to describe space-time by a metric family rather than by a single Lorentzian metric field?

The answer to the first question should be clear from the discussion in the previous sections. That is, by definition the role of t in the metric families is to describe global scale changes of the FHSs as measured by the FOs. This means that t should enter each metric family explicitly as a spatial scale factor $R(t)$. To avoid introducing some extra arbitrary scale or parameter we just define $R(t) = ct$. (Further justification of this choice of scale factor will be given later in this section.) Moreover the FHSs are by definition compact to ensure the uniqueness of a global time coordinate. That is, by requiring the FHSs to be compact we ensure that t splits quasi-metric space-time into a unique set of FHSs. Besides, since there is no reason to introduce any nontrivial spatial topology, the global basic geometry of the FHSs (neglecting the effects of gravity) should be that of the 3-sphere $\mathbf{S}^3$.

But any restriction on the global geometry of the FHSs implies the existence of prior 3-geometry. To prevent the possibility that the existence of prior 3-geometry may interfere with the dynamics of $\bar{\mathbf{g}}_t$, the family $\bar{\mathbf{g}}_t$ cannot be arbitrary but should take an even more restricted form than would seem necessary from the above requirements. Now a sensible restriction of $\bar{\mathbf{g}}_t$ would follow from the requirement that gravity should contain only one dynamical degree of freedom (i.e., that gravity is essentially *scalar*) in $(\mathcal{N}, \bar{\mathbf{g}}_t)$. This requirement is fulfilled if, expressed in a GTCS, the most general form allowed for the family $\bar{\mathbf{g}}_t$ is represented by the family of line elements (this may be taken as a definition)

$$\overline{ds}_t^2 = \Big[\bar{N}_s\bar{N}^s - \bar{N}_t^2\Big](dx^0)^2 + 2\frac{t}{t_0}\bar{N}_i dx^i dx^0 + \frac{t^2}{t_0^2}\bar{N}_t^2 S_{ik} dx^i dx^k, \tag{12}$$

where $\bar{N}_t$ is the lapse function field family and where $\frac{t_0}{t}\bar{N}^k\frac{\partial}{\partial x^k}$ is the family of shift vector fields in $\bar{\mathbf{g}}_t$. Moreover $S_{ik}dx^i dx^k$ is the metric of $\mathbf{S}^3$ (with radius equal to ct_0) and $\bar{N}_i \equiv \bar{N}_t^2 S_{ik}\bar{N}^k$. Note that whenever local conservation of energy and momentum holds, $\bar{N}_t$ should not depend on t in $(\mathcal{N}, \bar{\mathbf{g}}_t)$ but that it may nevertheless depend explicitly on the quantity $\frac{x^0}{ct}$. (Violation of local conservation laws is expected to occur in the early

Universe implying that $\bar{N}_t$ will depend on t in $(\mathcal{N}, \bar{\mathbf{g}}_t)$, see section 4.) On the other hand, a counterpart to the condition (2) in $(\mathcal{N}, \bar{\mathbf{g}}_t)$ implies that $\bar{N}^k$ cannot depend on t. Besides we notice that the form (12) of $\bar{\mathbf{g}}_t$ is preserved only under coordinate transformations between GTCSs. Also note that the most general allowed metric approximation of $\bar{\mathbf{g}}_t$ is the single metric $\bar{\mathbf{g}}$ obtained from equation (12) by setting $\frac{t}{t_0} = 1$ and replacing the metric of the 3-sphere with an Euclidean 3-metric.

As mentioned earlier, to get the correct affine structure on $(\mathcal{N}, \mathbf{g}_t)$ one *must* separate between ct and x^0 in $\mathbf{g}_t$. Similarly, to get the correct affine structure on $(\mathcal{N}, \bar{\mathbf{g}}_t)$, one must separate between ct and x^0 in equation (12). But the possibility that $\bar{N}_t$ depends explicitly on t means that the affine structure on $(\mathcal{N}, \bar{\mathbf{g}}_t)$ will differ slightly from that on $(\mathcal{N}, \mathbf{g}_t)$. That is, since counterparts to equations (2) and (5) must exist in $(\mathcal{N}, \bar{\mathbf{g}}_t)$, the t-dependence of $\bar{N}_t$ implies that the degenerate connection coefficients in $(\mathcal{N}, \bar{\mathbf{g}}_t)$ will not take a form exactly like that shown in equation (3). Rather, the in general non-vanishing connection coefficients in $(\mathcal{N}, \bar{\mathbf{g}}_t)$ are given in a GTCS by (a comma denotes partial derivation)

$$\stackrel{\star}{\bar{\Gamma}}{}^{0}_{t0} = \frac{\bar{N}_{t,t}}{\bar{N}_t}, \qquad \stackrel{\star}{\bar{\Gamma}}{}^{i}_{tj} = (\frac{1}{t} + \frac{\bar{N}_{t,t}}{\bar{N}_t})\delta^i_j, \qquad \stackrel{\star}{\bar{\Gamma}}{}^{\alpha}_{t\mu} \equiv \stackrel{\star}{\bar{\Gamma}}{}^{\alpha}_{\mu t}, \qquad \stackrel{\star}{\bar{\Gamma}}{}^{\alpha}_{\nu\mu} \equiv \bar{\Gamma}^{\alpha}_{(t)\nu\mu}. \tag{13}$$

Next we want to describe the evolution of the spatial scale factor $\bar{F}_t \equiv \bar{N}_t ct$ of the FHSs in the hypersurface-orthogonal direction. By definition we have (the symbol '$\bar{\perp}$' denotes a scalar product with $-\bar{\mathbf{n}}_t$)

$$\bar{F}_t^{-1} \stackrel{\star}{\pounds}_{\bar{\mathbf{n}}_t} \bar{F}_t \equiv \bar{F}_t^{-1}\Big((c\bar{N}_t)^{-1}\bar{F}_{t,t} + \pounds_{\bar{\mathbf{n}}_t}\bar{F}_t\Big) = (c\bar{N}_t)^{-1}\Big[\frac{1}{t} + \frac{\bar{N}_{t,t}}{\bar{N}_t}\Big] - \frac{\bar{N}_{t,\bar{\perp}}}{\bar{N}_t} \equiv c^{-2}\bar{x}_t + c^{-1}\bar{H}_t, \tag{14}$$

where $\pounds_{\bar{\mathbf{n}}_t}$ denotes Lie derivation in the direction normal to the FHS, treating t as a constant where it occurs explicitly. In equation (14) $c^{-2}\bar{x}_t$ represents the *kinematical contribution* to the evolution of $\bar{F}_t$ and $c^{-1}\bar{H}_t$ represents the so-called *non-kinematical contribution* defined by

$$\bar{H}_t \equiv \frac{1}{\bar{N}_t t} + \bar{y}_t, \qquad \bar{y}_t \equiv c^{-1}\sqrt{\bar{a}_{\mathcal{F}k}\bar{a}^k_{\mathcal{F}}}, \qquad c^{-2}\bar{a}_{\mathcal{F}j} \equiv \frac{\bar{N}_{t,j}}{\bar{N}_t}, \tag{15}$$

where $\bar{\mathbf{a}}_{\mathcal{F}}$ is the 4-acceleration field of the FOs in the family $\bar{\mathbf{g}}_t$. We see that the non-kinematical evolution (NKE) of the spatial scale factor takes the form of an "expansion" since $\bar{H}_t$ can never take negative values. Furthermore we observe that $\bar{H}_t$ does not vanish even if the kinematical evolution of $\bar{F}_t$ does and $\bar{N}_t$ is a constant in $(\mathcal{N}, \bar{\mathbf{g}}_t)$. For this particular case (see section 4, equation (44) below) we have the relationship $\bar{H}_t = \frac{1}{\bar{N}_t t} = \sqrt{\frac{\bar{P}_t}{6}}c$, where $\bar{P}_t$ is the Ricci scalar curvature intrinsic to the FHSs. *This*

means that in quasi-metric relativity, a global increase in scale of the FHSs is linked to their global curvature. Moreover this global increase of scale has nothing to do with the kinematical structure described by any single Lorentzian metric field. It follows that in QMR, the Hubble law is not interpreted as a kinematical law, rather the Hubble law is interpreted as evidence for global spatial curvature. This reinterpretation of the Hubble law also justifies the choice of scale factor $R(t) = ct$ since no other choice fulfills the above relationship with $\bar{H}_t$ playing the role of a "Hubble parameter" for the special case when $\bar{N}_t$ is a constant. In particular, notice that it is not possible to construct similar models where the global NKE takes the form of a "contraction" without introducing some extra arbitrary scale.

It follows from the above discussion that quasi-metric space-time is manifestly time-asymmetric by construction, irrespective of the fact that dynamical laws are time-reversal invariant. That is, quasi-metric space-time is time-asymmetric regardless of whether solutions of dynamical equations are time-symmetric or not. For example, one may find time-symmetric (e.g. static) solutions for $\bar{N}_t$ from equation (12). But the scale factor is never time-symmetric, as can be seen from equation (14). This illustrates that the global time-asymmetry of quasi-metric space-time is due to the cosmological arrow of time represented by the global cosmic expansion. Moreover, the existence of a global arrow of time means that quasi-metric space-time has a simple causal structure.

An interpretation of the Hubble law as a direct consequence of the Universe's global spatial curvature is impossible in metric theory. In fact, as mentioned in the introduction the possibility of finding an alternative description of the cosmic expansion was part of the physical motivation for constructing QMR in the first place. That is, in metric theory a wide variety of cosmological models (and in particular time-symmetric ones) are possible in principle, and which one happens to describe our Universe is not deducible from first principles. As a consequence the predictive power of metric theory is rather weak when it comes to cosmology. The main reason for this is that the expansion history of the Universe does not follow from first principles in metric theory since it depends on (arbitrary) cosmic initial conditions and the corresponding solutions of dynamical field equations.

The answer to the second question we posed above is now clear. In order to construct a new theory with considerably more predictive power than metric theory in cosmology, it would seem necessary to describe the cosmic expansion as non-kinematical, i.e., as some sort of prior geometric property of space-time itself. In this way an enormous multitude of possibilities regarding cosmic genesis and evolution will be eliminated just by postulating that space-time is quasi-metric. This is why quasi-metric space-time should be preferred

over Lorentzian space-time as a matter of principle as long as this position is not in conflict with observations.

3.2 Units and measurement

The fact that QMR describes global scale changes of the FHSs as non-kinematical suggests the existence of two fundamentally different scales in the Universe, one gravitational and one atomic. This means that we have to specify which kind of units we are supposed to use in equation (12). In metric theory it does not matter which kind of units one uses, but in quasi-metric theory this is not so obvious. That is, is equation (12) equally valid in units operationally defined from systems where gravitational interactions are dominant, as in operationally defined atomic units based on systems where gravitational interactions are insignificant? It turns out that the answer to this question is negative.

The units implicitly assumed when writing down line elements of the type (12) should be "atomic" units; i.e., units operationally defined by using atomic clocks and rods only. This means that we may interpret the variation in space-time of the spatial scale factor $\bar{F}_t$ as a consequence of the fact that we use atomic units to measure gravitational scales. Equivalently we may interpret the variation of $\bar{F}_t$ to mean that by definition, operationally defined atomic units are considered formally variable throughout space-time. (This interpretation is possible since any non-local inter-comparison of operationally defined units is purely a matter of definition.) The formal variation of atomic units in space-time means that gravitational quantities get an extra formal variation when measured in atomic units (and vice versa). (This shows up explicitly e.g. in differential laws such as local conservation laws.) We now postulate that atomic units vary in space-time just as the inverse of the spatial scale factor $\bar{F}_t$ since this implies that the scale of the FHSs does not vary measured in gravitational units. That is, any gravitational quantity gets a formal variability as some power of $\bar{F}_t$ when measured in atomic units. By definition c and Planck's constant $\hbar$ are not formally variable (this yields no physical restrictions since c and $\hbar$ cannot be combined to get a dimensionless number). This means that the formal variation of atomic length and atomic time units are identical and inverse to that of atomic energy (or mass) units. As a consequence, according to QMR the gravitational coupling parameter G_t is not a constant measured in atomic units. But note that since $\bar{F}_t$ is a constant in the Newtonian limit, Newtonian theory with a formally variable G_t is inconsistent with the Newtonian limit of QMR.

By dimensional analysis it is found that G_t varies as coordinate length squared measured in atomic units (i.e., as $\bar{F}_t^2$). But since G_t usually occurs in combination with

masses it is convenient to define G_t to take a constant value G and rather separate between *active mass* m_t (measured dynamically as a source of gravity) and *passive mass* m (i.e., passive gravitational mass or inertial mass). That is, we include the formal variation of G_t into m_t, which means that the formal variation of active mass goes as $\bar{F}_t$. This implies that the formal variation of the active stress-energy tensor $\mathbf{T}_t$ (considered as a source of gravitation) goes as $\bar{F}_t^{-2}$. We thus have

$$m_t = \frac{\bar{F}_t}{\bar{F}_{t_0}} m_{t_0} = \frac{\bar{N}_t t}{\bar{N}_0 t_0} m_0, \tag{16}$$

where $\bar{N}_0$ and m_0 denote values at some arbitrary reference event. (Formal variations of other gravitational quantities may be found similarly.) By convention we choose $\bar{N}_0 = 1$; this means that the (hypothetical) reference situation is an empty Universe at epoch t_0 (see section 4).

Note that the necessity to separate between gravitational and atomic scales represents a violation of the Strong Equivalence Principle (SEP).

3.3 Field equations

In metric theory there are no obstacles to having a full coupling between space-time geometry and the stress-energy tensor $\mathbf{T}$. In fact a full coupling is desirable since a partial coupling would result in so-called "prior" geometry, i.e., non-dynamical aspects of the space-time geometry which are not influenced by matter sources. On the other hand, in QMR we have already restricted the metric family $\bar{\mathbf{g}}_t$ by requiring that it takes the form (12). We thus have prior geometry, and it is not difficult to see that the prior geometry is a direct consequence of the particular form of the spatial geometry postulated in (12). But contrary to metric theory, in quasi-metric theory this kind of prior geometry is *necessary* since it makes possible global scale changes of the FHSs due to the NKE. From equation (15) we see that global scale changes of the FHSs come from the the global part of the spatial curvature (obtained by setting $\bar{N}_t$ =constant on each FHS), so by definition it has nothing to do with gravity. It is thus reasonable to require that *the intrinsic curvature of the FHSs should not couple explicitly to matter sources in quasi-metric gravity.*

This means that we must look for field equations which represent a *partial* coupling between space-time geometry and $\mathbf{T}_t$, where the geometrical quantities involved should not depend explicitly on the intrinsic curvature of the FHSs. Furthermore we should have metric correspondence with Newtonian theory in a natural, geometrical way. Fortunately such a correspondence having all the wanted properties exists already in GR, yielding a

natural correspondence with GR as well. That is, we postulate one field equation valid for projections with respect to the FHSs, namely (using a GTCS)

$$2\bar{R}_{(t)\bar{\perp}\bar{\perp}} = 2(c^{-2}\bar{a}^{i}_{\mathcal{F}|i} + c^{-4}\bar{a}_{\mathcal{F}i}\bar{a}^{i}_{\mathcal{F}} - \bar{K}_{(t)ik}\bar{K}^{ik}_{(t)} + \pounds_{\bar{\mathbf{n}}_t}\bar{K}_t) = \kappa(T_{(t)\bar{\perp}\bar{\perp}} + \hat{T}^{i}_{(t)i}), \tag{17}$$

where $\bar{\mathbf{R}}_t$ is the Ricci tensor family and $\bar{\mathbf{K}}_t$ is the extrinsic curvature tensor family (with trace $\bar{K}_t$) of the FHSs. Moreover $\kappa\equiv 8\pi G/c^4$, a "hat" denotes an object projected into the FHSs and the symbol '|' denotes spatial covariant derivation. The value of G is by convention chosen as that measured in a (hypothetical) local gravitational experiment in an empty Universe at epoch t_0. Note that $\bar{\mathbf{a}}_{\mathcal{F}}$ is an object intrinsic to the FHSs. Also note that all quantities correspond to the metric family $\bar{\mathbf{g}}_t$.

A second set of field equations having the desirable properties of not being explicitly dependent on spatial curvature in addition to yielding a natural correspondence with GR is (in a GTCS)

$$\bar{R}_{(t)j\bar{\perp}} = \bar{K}^{i}_{(t)j|i} - \bar{K}_{t,j} = \kappa T_{(t)j\bar{\perp}}, \tag{18}$$

again valid for projections with respect to the FHSs. Superficially, the field equations (17) and (18) look just as a subset of the Einstein field equations in ordinary GR. *But the crucial difference is that (17) and (18) are valid only for projections with respect to the FHSs; they do not hold for projections with respect to any other hypersurfaces.* Contrary to this, in GR metric counterparts to equations (17) and (18) hold for projections with respect to arbitrary hypersurfaces; this is a direct result of the Einstein field equations. Notice that the field equations (17), (18) are time-reversal invariant.

We now have a sufficient number of field equations to determine the unknown quantities $\bar{N}_t$ and $\bar{N}_j$ in equation (12). Besides we observe that equation (17) represents one dynamical equation whereas equations (18) are constraints. To illustrate some general properties of the field equations it is useful to have an explicit expression for $\bar{\mathbf{K}}_t$, which may be calculated from equation (12). Using a GTCS we find

$$\bar{K}_{(t)ij} = \frac{t}{2t_0\bar{N}_t}(\bar{N}_{i|j} + \bar{N}_{j|i}) + \Big(\frac{\bar{N}_{t,\bar{\perp}}}{\bar{N}_t} - \frac{t_0}{t}c^{-2}\bar{a}_{\mathcal{F}k}\frac{\bar{N}^k}{\bar{N}_t}\Big)\bar{h}_{(t)ij}, \tag{19}$$

$$\bar{K}_t = \frac{t_0}{t}\frac{\bar{N}^{i}{}_{|i}}{\bar{N}_t} + 3\Big(\frac{\bar{N}_{t,\bar{\perp}}}{\bar{N}_t} - \frac{t_0}{t}c^{-2}\bar{a}_{\mathcal{F}k}\frac{\bar{N}^k}{\bar{N}_t}\Big), \tag{20}$$

where $\bar{\mathbf{h}}_t$ is the metric family intrinsic to the FHSs in $(\mathcal{N}, \bar{\mathbf{g}}_t)$. Note that it is convenient to study systems where equations (17), (18) and (19) can be simplified. That is, to simplify calculations it useful to consider systems where the condition

$$\bar{N}^{i}{}_{|i} = 3c^{-2}\bar{a}_{\mathcal{F}i}\bar{N}^i, \qquad \Rightarrow \qquad \bar{K}_t = 3\frac{\bar{N}_{t,\bar{\perp}}}{\bar{N}_t}, \tag{21}$$

holds. For example, by substituting equations (19) and (21) into equation (17) we easily see the reason why $\bar{N}_t$ in general must depend on t; namely that the various terms in equation (17) scale differently with respect to the factor $\frac{t_0}{t}$. Note that even if the quantity $\bar{N}_{t,\bar{\perp}}$ does get some extra variability on the FHSs due to the explicit dependence of $\bar{N}_t$ on t, this does not necessarily apply to $\bar{N}_t$ itself.

It is also convenient to have explicit expressions for the curvature intrinsic to the FHSs. From equation (12) one easily calculates

$$\bar{H}_{(t)ij} = c^{-2}\Big(\bar{a}^{k}_{\mathcal{F}|k} - \frac{1}{\bar{N}_t^2 t^2}\Big)\bar{h}_{(t)ij} - c^{-4}\bar{a}_{\mathcal{F}i}\bar{a}_{\mathcal{F}j} - c^{-2}\bar{a}_{\mathcal{F}i|j}, \tag{22}$$

$$\bar{P}_t = \frac{6}{(\bar{N}_t ct)^2} + 2c^{-4}\bar{a}_{\mathcal{F}k}\bar{a}^{k}_{\mathcal{F}} - 4c^{-2}\bar{a}^{k}_{\mathcal{F}|k}, \tag{23}$$

where $\bar{\mathbf{H}}_t$ is the Einstein tensor family intrinsic to the FHSs in $(\mathcal{N}, \bar{\mathbf{g}}_t)$.

3.4 Local conservation laws

Within the metric framework one usually just substitutes partial derivatives with covariant derivatives when generalizing differential laws from flat to curved space-time. In fact this rule in the form "comma goes to semicolon" follows directly from the EEP in most metric theories [1]. But in quasi-metric theory it is possible to couple non-gravitational fields to first derivatives of the scale factor of the FHSs such that the EEP still holds. That is, any coupling of non-gravitational fields to the fields $\bar{a}_{\mathcal{F}j}$, $\frac{\bar{N}_{t,\bar{\perp}}}{\bar{N}_t}$, $\frac{\bar{N}_{t,t}}{\bar{N}_t}$ and $\frac{1}{t}$ may be made to vanish in the local inertial frames so that these couplings do not interfere with the local non-gravitational physics.

In particular the EEP implies that the local conservation laws take the form $\nabla{\cdot}\mathbf{T} = 0$ in any metric theory based on an invariant action principle, independent of the field equations [1]. The reason why the conservation laws must take this form is that they then imply that inertial test particles move on geodesics of the metric. So, in said metric theories the above form of the local conservation laws is sufficient to ensure that they are consistent with the equations of motion. But in quasi-metric theory, consistency with the equations of motion does not necessarily imply that the local conservation laws take the form shown above. This fact, in addition to the possibility of extra couplings between non-gravitational fields and the fields $\bar{a}_{\mathcal{F}j}$, $\frac{\bar{N}_{t,\bar{\perp}}}{\bar{N}_t}$, $\frac{\bar{N}_{t,t}}{\bar{N}_t}$ and $\frac{1}{t}$, means that the EEP does not necessarily imply a form similar to $\nabla{\cdot}\mathbf{T} = 0$ of the local conservation laws in quasi-metric theory. That is, the divergence $\overset{\star}{\bar{\nabla}}{\cdot}\mathbf{T}_t$ will in general not vanish, so the EEP is insufficient to determine the form of the local conservation laws in QMR.

Since the EEP is not sufficient to determine the form of the local conservation laws in quasi-metric theory we have to deduce their form from other criteria. To do that we first write down the coordinate expression for $\bar{\overset{\star}{\nabla}}\cdot\mathbf{T}_t$. This reads

$$T^{\nu}_{(t)\mu\bar{*}\nu} \equiv T^{\nu}_{(t)\mu;\nu} + c^{-1}T^{0}_{(t)\mu\bar{*}t}, \tag{24}$$

where the symbol '$\bar{*}$' denotes degenerate covariant derivation compatible with the family $\bar{\mathbf{g}}_t$ and a semicolon denotes metric covariant derivation in component notation. It is straightforward to calculate the second term on the right hand side of equation (24), and assuming that the only t-dependence of $\mathbf{T}_t$ is via the formally variable units we find

$$T^{0}_{(t)\mu\bar{*}t} = -\frac{2}{\bar{N}_t}\Big(\frac{1}{t} + \frac{\bar{N}_{t,t}}{\bar{N}_t}\Big)T_{(t)\bar{\perp}\mu}. \tag{25}$$

Second, in order to have the correct Newtonian limit in addition to being consistent with electromagnetism coupled to gravity [4], the first term on the right hand side of equation (24) must take the form

$$T^{\nu}_{(t)\mu;\nu} = 2\frac{\bar{N}_{t,\nu}}{\bar{N}_t}T^{\nu}_{(t)\mu} = 2c^{-2}\bar{a}_{\mathcal{F}i}\hat{T}^{i}_{(t)\mu} - 2\frac{\bar{N}_{t,\bar{\perp}}}{\bar{N}_t}T_{(t)\bar{\perp}\mu}. \tag{26}$$

By applying (26) to a source consisting of a perfect fluid with no pressure (i.e., dust) and projecting the resulting equations with the quantity $\bar{\mathbf{g}}_t + c^{-2}\bar{\mathbf{u}}_t\otimes\bar{\mathbf{u}}_t$, we find that the dust particles move on geodesics of $\bar{\overset{\star}{\nabla}}$ in $(\mathcal{N}, \bar{\mathbf{g}}_t)$. This guarantees that the dust particles move on geodesics of $\overset{\star}{\nabla}$ in $(\mathcal{N}, \mathbf{g}_t)$ as well; see section 3.5 for justification. Besides, since $\mathbf{T}_t$ is the active stress-energy tensor and since no extra field independent of $\bar{\mathbf{g}}_t$ couples to gravitating bodies in QMR, this result should apply even to (sufficiently small) dust particles with significant self-gravitational energy and not only to test particles. This means that although G_t is formally variable in QMR, no Nordtvedt effect should be associated with this formal variability.

Since $\mathbf{T}_t$ is not directly measurable locally one must know how it relates to the *passive stress-energy tensor* $\mathcal{T}_t$ in $(\mathcal{N}, \mathbf{g}_t)$, or equivalently, to the passive stress-energy tensor $\bar{\mathcal{T}}_t$ in $(\mathcal{N}, \bar{\mathbf{g}}_t)$ (which can be measured locally using atomic units). This means that equations (26) do not represent the "more physical" local conservation laws involving $\mathcal{T}_t$. Besides the local conservation laws shown in (26) are compatible with $\bar{\mathbf{g}}_t$ and not with $\mathbf{g}_t$. However, said more physical local conservation laws are found by calculating $\overset{\star}{\nabla}\cdot\mathcal{T}_t$ when $\mathbf{g}_t$ is known. But these more physical local conservation laws take no predetermined form.

The relationship between $\mathbf{T}_t$ and $\mathcal{T}_t$ (or $\bar{\mathcal{T}}_t$) depends in principle explicitly on the general nature of the matter source. For example, this relationship will be different for a

perfect fluid consisting of material point particles than for pure radiation. To illustrate this we may consider $\mathbf{T}_t$ for a perfect fluid:

$$\mathbf{T}_t = (\tilde{\rho}_{\rm m} + c^{-2}\tilde{p})\bar{\mathbf{u}}_t \otimes \bar{\mathbf{u}}_t + \tilde{p}\bar{\mathbf{g}}_t, \tag{27}$$

where $\tilde{\rho}_{\rm m}$ is the active mass-energy density in the local rest frame of the fluid and $\tilde{p}$ is the active pressure. The corresponding expressions for $\mathcal{T}_t$ and $\bar{\mathcal{T}}_t$ are

$$\mathcal{T}_t = \sqrt{\frac{\bar{h}_t}{h_t}}\Big[(\rho_{\rm m} + c^{-2}p)\mathbf{u}_t \otimes \mathbf{u}_t + p\mathbf{g}_t\Big], \qquad \bar{\mathcal{T}}_t = (\rho_{\rm m} + c^{-2}p)\bar{\mathbf{u}}_t \otimes \bar{\mathbf{u}}_t + p\bar{\mathbf{g}}_t, \tag{28}$$

where $\rho_{\rm m}$ is the passive mass-energy density as measured in the local rest frame of the fluid and p is the passive pressure. Also, by definition $\bar{h}_t$ and h_t are the determinants of $\bar{\mathbf{h}}_t$ and $\mathbf{h}_t$, respectively. Now the relationship between $\tilde{\rho}_{\rm m}$ and $\rho_{\rm m}$ is given by

$$\rho_{\rm m} = \begin{cases} \frac{t_0}{t}\bar{N}_t^{-1}\tilde{\rho}_{\rm m}, & \text{for a fluid of material point particles,} \\ \frac{t_0^2}{t^2}\bar{N}_t^{-2}\tilde{\rho}_{\rm m}, & \text{for a null fluid,} \end{cases} \tag{29}$$

and a similar relationship exists between $\tilde{p}$ and p. The reason why the relationship between $\tilde{\rho}_{\rm m}$ and $\rho_{\rm m}$ is different for a null fluid than for other perfect fluid sources is that gravitational or cosmological spectral shifts of null particles influence their passive mass-energy but not their active mass-energy.

3.5 Constructing $\mathbf{g}_t$ from $\bar{\mathbf{g}}_t$

As mentioned previously the field equations (17), (18) contain only one dynamical degree of freedom coupled explicitly to matter. To see this it is convenient to choose a system where the condition (21) holds. Then $\bar{K}_t = 3\frac{\bar{N}_{t,\bar{\perp}}}{\bar{N}_t}$, and we see that the explicitly coupled dynamical field is the lapse function field $\bar{N}_t$. But from equation (15) we see that spatial derivatives of $\bar{N}_t$ yield local contributions to the NKE of the FHSs as well. *These local contributions are not realized explicitly in the evolution of $\bar{F}_t$, so whenever $\bar{y}_t \neq 0$ in equation (15), it is necessary to construct a new metric family $\mathbf{g}_t$.* In the following we will see the reason why.

The question now is just how the metric family (12) should be modified to include the local effects of the NKE. This question can be answered by noticing that according to equation (15), the local effects of the NKE should take the form of an "expansion" that varies from place to place. That is, the tangent spaces of the FHSs should experience a varying degree of expansion as a consequence of the local contribution $\bar{y}_t$ to $\bar{H}_t$. Two points now are that the local contribution $\bar{y}_t$ to the expansion is due to gravitation

and that this contribution is not reflected explicitly in the evolution of the scale factor $\bar{F}_t$ as can be seen from equation (14). Thus, whenever $\bar{y}_t$ is nonzero we may think of change of distances in any tangent space of the FHSs as consisting of an expansion plus a contraction. That is, the FOs seem to "move" more than the explicit change of $\bar{F}_t$ should indicate. The modification of the family (12) then consists of a compensation for this extra gravitationally induced "motion".

To have a consistent transformation $\bar{\mathbf{g}}_t \rightarrow \mathbf{g}_t$ we need to treat the effects of the extra gravitationally induced "motion" in each tangent space, i.e., locally. To do that it turns out that we need to define a 3-vector field $\mathbf{b}_{\mathcal{F}}$ representing the coordinate distance to a local fictitious "center of gravity" in each tangent space. This is necessary to be able to define a family of 3-vector fields $\mathbf{v}_t$ telling how much the FOs in each tangent space "recede" from the local "center of gravity" due to the gravitationally induced expansion. Besides, since the coordinate positions of all FOs must be unaffected, the FOs must simultaneously "fall" with velocity $-\mathbf{v}_t$ toward the local "center of gravity" to cancel out the "recession". And the extra "motion" involved induces corrections in the coordinate length and time intervals as perceived by any FO. That is, the metric components of (12) in a GTCS must be modified to yield a new metric family $\mathbf{g}_t$.

In the special case where $\bar{\mathbf{g}}_t$ is spherically symmetric with respect to one distinguished point, the spatial coordinates of this point represent a natural local "center of gravity" in each tangent space of the FHSs. In this case we obviously have $\mathbf{b}_{\mathcal{F}} = r\frac{\partial}{\partial r}$ expressed in a spherical polar GTCS where the distinguished point lies at the origin. It seems reasonable to seek for an equation defining $\mathbf{b}_{\mathcal{F}}$ which yields this solution for the spherically symmetric case. Furthermore the wanted equation should be linear in $\mathbf{b}_{\mathcal{F}}$ to ensure unique solutions, and it should involve $\bar{\mathbf{a}}_{\mathcal{F}}$ since any deviation from spherical symmetry will be encoded into $\bar{\mathbf{a}}_{\mathcal{F}}$ and its spatial derivatives. By inspection of the spherically symmetric case it turns out that it is possible to find an equation which has all the desired properties, namely

$$\Big[\bar{a}^k_{\mathcal{F}|k} + c^{-2}\bar{a}_{\mathcal{F}k}\bar{a}^k_{\mathcal{F}}\Big]b^j_{\mathcal{F}} - \Big[\bar{a}^j_{\mathcal{F}|k} + c^{-2}\bar{a}_{\mathcal{F}k}\bar{a}^j_{\mathcal{F}}\Big]b^k_{\mathcal{F}} - 2\bar{a}^j_{\mathcal{F}} = 0. \tag{30}$$

With $\mathbf{b}_{\mathcal{F}}$ defined from (30) we are now able to define the 3-vector family $\mathbf{v}_t$. Expressed in a GTCS the vector field family $\mathbf{v}_t$ by definition has the components [2]

$$v^j_{(t)} \equiv \bar{y}_t b^j_{\mathcal{F}}, \qquad v = \bar{y}_t\sqrt{\bar{h}_{(t)ik}b^i_{\mathcal{F}}b^k_{\mathcal{F}}}, \tag{31}$$

where v is the norm of $\mathbf{v}_t$. Note that v does not depend explicitly on t except via the possible t-dependence of $\bar{N}_t$.

Now $\mathbf{g}_t$ is constructed algebraically from $\bar{\mathbf{g}}_t$ and v. To do that we first include the effects of the gravitationally induced expansion as seen from new observers which do not experience this extra expansion, they are by definition "at rest". This yields a correction to spatial intervals in the $\mathbf{b}_{\mathcal{F}}$-direction due to the radial Doppler effect, the correction factor being $\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{1/2}$. There is also an inverse time dilation correction factor $(1-\frac{v^2}{c^2})^{1/2}$ to coordinate time intervals. There are no correction factors for spatial intervals normal to the $\mathbf{b}_{\mathcal{F}}$-direction. Second, the coordinate intervals for the said new observers get an identical pair of correction factors when compared to observers (now by definition "at rest") moving with relative velocity $-\mathbf{v}_t$.

To define transformation formulae it is convenient to define the unit vector field $\bar{\mathbf{e}}_b \equiv \frac{t_0}{t}\bar{e}^i_b\frac{\partial}{\partial x^i}$ and the corresponding covector field $\bar{\mathbf{e}}^b \equiv \frac{t}{t_0}\bar{e}^b_i dx^i$ along $\mathbf{b}_{\mathcal{F}}$. Since it is in general not possible or practical to construct a GTCS where $\bar{\mathbf{e}}_b$ is parallel to one of the coordinate vector fields one expects that the transformation formulae defining the transformation $\bar{\mathbf{g}}_t \rightarrow \mathbf{g}_t$ should involve components of $\bar{\mathbf{e}}^b$. Requiring correspondence with the spherically symmetric case one finds [2]

$$g_{(t)00} = \left(1-\frac{v^2}{c^2}\right)^2 \bar{g}_{(t)00}, \tag{32}$$

$$g_{(t)0j} = \left(1-\frac{v^2}{c^2}\right)\Big[\bar{g}_{(t)0j} + \frac{t}{t_0}\frac{2\frac{v}{c}}{1-\frac{v}{c}}(\bar{e}^i_b\bar{N}_i)\bar{e}^b_j\Big], \tag{33}$$

$$g_{(t)ij} = \bar{g}_{(t)ij} + \frac{t^2}{t_0^2}\frac{4\frac{v}{c}}{(1-\frac{v}{c})^2}\bar{e}^b_i\bar{e}^b_j. \tag{34}$$

Notice that we have eliminated any possible t-dependence of $\bar{N}_t$ in equations (32)-(34) by setting $t = x^0/c$ where it occurs. This implies that N does not depend explicitly on t.

Some tensor fields are required to preserve their norm under the transformation defined in equations (32)-(34). One particular example of this is the transformation $\bar{\mathbf{F}}_t \rightarrow \mathbf{F}_t$ of the passive electromagnetic field tensor family $\bar{\mathbf{F}}_t$ in $(\mathcal{N}, \bar{\mathbf{g}}_t)$ to its counterpart $\mathbf{F}_t$ in $(\mathcal{N}, \mathbf{g}_t)$, where the latter enters into the Lorentz force law [4]. This suggests that the transformation $\bar{\mathbf{g}}_t \rightarrow \mathbf{g}_t$ defined in equations (32)-(34) is only a particular case of a more general transformation. That is, transformations similar to $\bar{\mathbf{g}}_t \rightarrow \mathbf{g}_t$ should apply to any tensor field which norm is required to be unchanged when $\bar{\mathbf{g}}_t \rightarrow \mathbf{g}_t$. As an example we list the formulae defining the transformation $\bar{\mathbf{Z}}_t \rightarrow \mathbf{Z}_t$ where $\bar{\mathbf{Z}}_t$ is a rank one tensor field family. ($\bar{\mathbf{Z}}_t$ may, e.g., be identified with a general 4-velocity vector family $\bar{\mathbf{u}}_t$.) These formulae read

$$Z_{(t)0} = \left(1-\frac{v^2}{c^2}\right)\bar{Z}_{(t)0}, \qquad Z^0_{(t)} = \left(1-\frac{v^2}{c^2}\right)^{-1}\bar{Z}^0_{(t)}, \tag{35}$$

$$Z_{(t)j}=\bar{Z}_{(t)j}+\frac{2\frac{v}{c}}{1-\frac{v}{c}}(\bar{e}^i_b\bar{Z}_{(t)i})\bar{e}^b_j,\qquad \hat{Z}^j_{(t)}=\hat{\bar{Z}}^j_{(t)}-\frac{2\frac{v}{c}}{1+\frac{v}{c}}(\bar{e}^b_i\hat{\bar{Z}}^i_{(t)})\bar{e}^j_b. \tag{36}$$

It is possible to find similar transformation formulae valid for higher rank tensor field families. For illustrative purposes we also list formulae valid for the transformation $\bar{\mathbf{Q}}_t\rightarrow\mathbf{Q}_t$, where $\bar{\mathbf{Q}}_t$ is a rank two tensor field family. These formulae read

$$Q_{(t)00}=\Big(1-\frac{v^2}{c^2}\Big)^2\bar{Q}_{(t)00},\qquad Q^{00}_{(t)}=\Big(1-\frac{v^2}{c^2}\Big)^{-2}\bar{Q}^{00}_{(t)}, \tag{37}$$

$$\begin{aligned} Q_{(t)0j} &= \Big(1-\frac{v^2}{c^2}\Big)\Big[\bar{Q}_{(t)0j}+\frac{2\frac{v}{c}}{1-\frac{v}{c}}(\bar{e}^i_b\bar{Q}_{(t)0i})\bar{e}^b_j\Big],\\ \hat{Q}^{0j}_{(t)} &= \Big(1-\frac{v^2}{c^2}\Big)^{-1}\Big[\hat{\bar{Q}}^{0j}_{(t)}-\frac{2\frac{v}{c}}{1+\frac{v}{c}}(\bar{e}^b_i\hat{\bar{Q}}^{0i}_{(t)})\bar{e}^j_b\Big], \end{aligned} \tag{38}$$

$$\begin{aligned} Q_{(t)ij} &= \bar{Q}_{(t)ij}+\frac{2\frac{v}{c}}{(1-\frac{v}{c})^2}\bar{e}^k_b(\bar{e}^b_i\bar{Q}_{(t)kj}+\bar{Q}_{(t)ik}\bar{e}^b_j),\\ \hat{Q}^{ij}_{(t)} &= \hat{\bar{Q}}^{ij}_{(t)}-\frac{2\frac{v}{c}}{(1+\frac{v}{c})^2}\bar{e}^b_k(\bar{e}^i_b\hat{\bar{Q}}^{kj}_{(t)}+\hat{\bar{Q}}^{ik}_{(t)}\bar{e}^j_b). \end{aligned} \tag{39}$$

These transformation formulae may easily be generalized to tensor field families of higher rank. Notice that it is also possible to find formulae for the inverse transformations $\mathbf{Z}_t\rightarrow\bar{\mathbf{Z}}_t$, $\mathbf{Q}_t\rightarrow\bar{\mathbf{Q}}_t$ and similarly for tensor field families of higher rank.

Since both $\bar{\mathbf{u}}_t$ and $\bar{\mathbf{F}}_t$ transform according to the rules (35)-(39), so must any 4-acceleration $\bar{\mathbf{a}}_t$ resulting from the Lorentz force acting on charged matter. This implies that the norm of $\bar{\mathbf{a}}_t$ must be invariant under the transformation. Since this result should apply to all 4-accelerations determined from non-gravitational forces and since such forces may in particular vanish, we may deduce that *geodesic motion in* $(\mathcal{N},\bar{\mathbf{g}}_t)$ *implies geodesic motion in* $(\mathcal{N},\mathbf{g}_t)$. That is, any inertial observer in $(\mathcal{N},\bar{\mathbf{g}}_t)$ must be inertial in $(\mathcal{N},\mathbf{g}_t)$ as well. Notice that the 4-acceleration field $\bar{\mathbf{a}}_{\mathcal{F}}$ does not in general transform according to the rules (35)-(36) since $\bar{\mathbf{a}}_{\mathcal{F}}$ is determined from the requirement that the FOs must move normal to the FHSs rather than from some non-gravitational force acting on the FOs.

By construction the family $\bar{\mathbf{g}}_t$ contains only one propagating dynamical degree of freedom. However the quantity v represents a second dynamical degree of freedom so one would expect that the number of propagating dynamical degrees of freedom in $(\mathcal{N},\mathbf{g}_t)$ is two, the same as in GR. Note that the dynamical degree of freedom represented by v is *implicit* inasmuch as it is not explicitly coupled to matter fields.

We close this section by emphasizing that $\mathbf{g}_t$ is the "physical" metric family in the sense that $\mathbf{g}_t$ should be used consequently when comparing predictions of QMR to experiments. That is, any laws given in terms of $\bar{\mathbf{g}}_t$ and its associated connection, e.g. the

local conservation laws defined in (26), are not the "physical" laws; those must always be in terms of $\mathbf{g}_t$ and its associated connection when comparing directly to experiment. Nevertheless it is sometimes necessary to use the laws in terms of $\bar{\mathbf{g}}_t$ and its associated connection. For example, to be able to calculate $\bar{\mathbf{g}}_t$ it is in general necessary to use the local conservation laws (26). But as long as one is aware of the correct relationship between laws and observables this should not represent any problem.

3.6 Comparing theory to experiment

To be able to compare the predictions of quasi-metric theory to experiment it would be useful to have some systematic weak field approximation method similar to the parametrized post-Newtonian (PPN) formalism developed for metric theories of gravity. It is not a good idea to try to apply the standard PPN-formalism to our quasi-metric theory, however. There are several reasons for this; one obvious reason is that the PPN-formalism neglects the non-metric aspects of QMR. This means that any PPN-analysis of our field equations is limited to their metric approximations. But even these metric approximations are not suitable for a standard PPN-analysis since the resulting PPN-metric $\bar{\mathbf{g}}$ describes only the explicit dynamical degree of freedom and so is not the one to which experiments are to be compared. This means that the PPN-metric $\bar{\mathbf{g}}$ will *not* have an acceptable set of PPN-parameters according to metric theory. For example, a PPN-analysis of our field equations yields the PPN-parameters $\gamma = -1$ and $\beta = 0$; both values are totally unacceptable for any viable metric theory. Moreover the differences between QMR and metric gravity regarding the implementation of the EEP show up via the local conservation laws (26) since even in the metric approximation, these laws are different from their counterparts in standard metric theory. This means that any constraints on the PPN-parameters deduced from integral conservation laws [1] will not necessarily hold in QMR.

Besides, when one attempts to construct a "physical" PPN-metric $\mathbf{g}$ from $\bar{\mathbf{g}}$ in the manner discussed in the previous section one gets more complications. In particular the isotropic PPN coordinate system is not invariant under the transformation $\bar{\mathbf{g}} \rightarrow \mathbf{g}$. That is, isotropic coordinates for the metric $\mathbf{g}$ are different from the isotropic coordinates one started out with in the first place when solving the field equations! The reason for this is, of course, that the construction of isotropic coordinates depends on the metric. But the main problem here is that the PPN-formalism does not tackle properly the construction of $\mathbf{g}$ from $\bar{\mathbf{g}}$. That is, the PPN-formalism exclusively handles explicit dynamical degrees of freedom and neglects the possible existence of implicit dynamical degrees of freedom.

Consequently, the PPN-metric $\mathbf{g}$ may contain terms which do not occur in the PPN-metric obtained from any metric theory with only explicit dynamical degrees of freedom. Thus the bottom line is that a standard PPN-analysis, even limited to metric approximations of QMR, will fail.

Thus the fact is that to be able to compare the predictions of our theory to gravitational experiments performed in the solar system in a satisfactory way, a separate weak-field expansion similar to the PPN-formalism should be developed. And since such a formalism is lacking at this point in time it is not yet clear whether or not quasi-metric theory is viable. However, if a separate formalism is developed it should have some correspondence with the PPN-formalism to answer this question (minimizing the need for reanalyzing weak-field experiments within the new framework). But we may still calculate specific solutions with high symmetry to get an idea how the quasi-metric theory compares to GR. In particular, in the metric approximation we may calculate the exact counterpart to the Schwarzschild case of GR. That is, in Schwarzschild coordinates the static, spherically symmetric vacuum solution of equation (17) is [2] (the t-labels are omitted everywhere when dealing with metric approximations)

$$\overline{ds}^2 = \Big(\sqrt{1+(\frac{r_{\rm s}}{2r})^2} - \frac{r_{\rm s}}{2r}\Big)^2\Big(-(dx^0)^2 + [1+(\frac{r_{\rm s}}{2r})^2]^{-1}dr^2\Big) + r^2 d\Omega^2, \tag{40}$$

where $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$ and

$$r_{\rm s} \equiv \frac{2MG}{c^2}, \qquad M \equiv c^{-2}\int\int\int \bar{N}(T_{\hat{\perp}\hat{\perp}} + \hat{T}^i_{\ i})d\bar{V}. \tag{41}$$

In (41) M is the dynamically measured mass of the source and the integration is taken over the FHS. (The particular form of M follows directly from equation (17) applied to the interior of a spherically symmetric, static source when extrapolated to the exterior solution given by equation (40).) Furthermore, since $\mathbf{b}_{\mathcal{F}} = r\frac{\partial}{\partial r}$ for the spherically symmetric case we easily find from equations (15) and (31) that

$$v = \frac{r_{\rm s}c}{2r\sqrt{1+(\frac{r_{\rm s}}{2r})^2}}. \tag{42}$$

Then using equations (32) and (34) we get

$$\begin{aligned} ds^2 &= -[1+(\frac{r_{\rm s}}{2r})^2]^{-2}\Big(\frac{r_{\rm s}}{2r} + \sqrt{1+(\frac{r_{\rm s}}{2r})^2}\Big)^{-2}(dx^0)^2 \\ &\quad +[1+(\frac{r_{\rm s}}{2r})^2]^{-1}\Big(\frac{r_{\rm s}}{2r} + \sqrt{1+(\frac{r_{\rm s}}{2r})^2}\Big)^2 dr^2 + r^2 d\Omega^2 \\ &= -(1 - \frac{r_{\rm s}}{r} + \frac{3}{8}\frac{r_{\rm s}^3}{r^3} + \cdots)(dx^0)^2 + (1 + \frac{r_{\rm s}}{r} + \frac{r_{\rm s}^2}{4r^2} + \cdots)dr^2 + r^2 d\Omega^2. \end{aligned} \tag{43}$$

We see that this metric has no event horizon and that it is consistent with the four "classical" solar system tests. Note that this consistency is due to the existence of the implicitly coupled dynamical degree of freedom represented by the scalar field v.

It is important to notice that the metric (43) is only a metric approximation yielding correspondences between QMR and GR. But we may go beyond the metric approximation and include the effects of the non-metric part of QMR in the spherically symmetric case. This is done in [2] and [5] where it is shown that the quasi-metric theory predicts that the size of the solar system increases according to the Hubble law, but in a way such that the trajectories of non-relativistic test particles are not unduly affected. However, this prediction has a number of observable consequences which are seen and in good agreement with QMR [5], [6]. In particular the prediction that the solar system expands according to the Hubble law provides a natural explanation [6] of the apparently "anomalous" acceleration of some distant spacecraft as inferred from radiometric data [7].

We conclude that even if the PPN-formalism does not apply to QMR and that this makes the predictions of QMR more difficult to test against experiment, some of the non-metric aspects of QMR seem to agree well with observations. This represents a challenge for GR and other metric theories just as much as the successes of GR represent a challenge for any alternative theory of gravity. But it is a mathematical fact that metric theories are unable to handle the non-metric aspects of QMR in a geometrical manner, thus making it impossible to calculate any of these effects from first principles in metric gravity.

4 Quasi-metric cosmology

4.1 General predictions

Cosmology as done in QMR is radically different from any possible approach to the subject based on a metric theory of gravity. The main reason for this is, of course, that in QMR the expansion of the Universe is not interpreted as a kinematical phenomenon. Rather, by construction the cosmic expansion is a prior-geometric property of quasi-metric space-time itself. This means that any concept of the Universe as a purely gravitationally dynamic system simply is not valid in QMR. Consequently many of the problems encountered in traditional cosmology do not exist in quasi-metric cosmology. For example, in QMR the expansion history of the Universe does not depend on its matter density, so there is no flatness problem. Due to the coasting expansion no horizon problem exists either, nor is there any need for a cosmological constant. Thus QMR

yields some cosmological predictions from first principles, without the extra flexibility represented by the existence of a set of cosmological parameters. In QMR there will be cosmological problems not encountered in metric gravity, however.

The lack of any sort of cosmic dynamics in QMR is realized mathematically by the fact that no quasi-metric counterparts to the Friedmann-Robertson-Walker (FRW) models exist [2]. However, one possible cosmological model with isotropic FHSs is the toy model given by the metric family

$$\begin{aligned} ds_t^2 = \overline{ds}_t^2 &= -(dx^0)^2 + (\frac{t}{t_0})^2\Big(\frac{dr^2}{1-\frac{r^2}{(ct_0)^2}} + r^2 d\Omega^2\Big) \\ &= -(dx^0)^2 + (ct)^2\Big(d\chi^2 + \sin^2\chi d\Omega^2\Big), \end{aligned} \tag{44}$$

which represents an empty universe. This is a family of $\mathbf{S}^3\times\mathbf{R}$ space-time metrics, and it is easy to check that it satisfies the field equations without sources and also equations (14), (15). Besides this empty model it is possible to have a toy cosmological model where the Universe is filled with an isotropic null fluid. In this case one finds solutions of the type $\bar{N}_t = \exp[-K\frac{(x^0)^2}{(ct)^2}]$ (where K is a constant depending on the fluid density) from equation (17). Such solutions also satisfy equation (26). But since $\bar{N}_t$ is constant on the FHSs in these models, we may transform the resulting $\mathbf{g}_t$ into the metric family shown in equation (44) by doing trivial re-scalings of the time coordinate $\bar{N}_t x^0 \to x^0$ and of the global time function $\bar{N}_t t \to t$. (Since $\bar{N}_t < 1$ this means that the proper time elapsed for any FO at a given epoch is smaller for the null fluid model than for the empty one.) It is also possible to find isotropic null fluid models where there is local creation of null particles. In such models $\bar{N}_t$ will depend on t in $(\mathcal{N}, \bar{\mathbf{g}}_t)$, and equation (25) is violated.

Now one peculiar aspect of QMR is that gravitationally bound bodies made of ideal gas and their associated gravitational fields are predicted to expand according to the Hubble law [2], [5]. That is, if hydrostatic instabilities due to the expansion may be neglected (there are no such instabilities for an ideal gas), measured in atomic units linear sizes within a gravitationally bound system increase as the scale factor, i.e., proportional to t. Note that this is valid even for the quantity $c^{-2}GM_t$ (where M_t is any active mass), which has the dimension of length. On the other hand it is a prediction of QMR that except for a global cosmic redshift not noticeable locally, the electromagnetic field is unaffected by the global cosmic expansion [4]. This means that there is no reason to expect that atoms or other purely quantum-mechanical systems should participate in the cosmic expansion. (To clarify the effect of the expansion on atoms it would be necessary to perform calculations involving quantum fields in quasi-metric space-time.)

A universe filled with an isotropic fluid consisting of material particles is not possible

in QMR. That is, for any toy model universe filled with a perfect fluid source described by an equation of state different from $\rho_{\rm m}c^2 = 3p$, $\bar{N}_t$ must necessarily vary in space as well as in time [2]. However the more relativistic the fluid, the closer its equation of state to that of a null fluid and the weaker the spatial dependence of $\bar{N}_t$ will be. This means that one expects the deviations from isotropy to be very small in the early Universe, and eventually to shrink to zero in the limit $t \rightarrow 0$. On the other hand one expects the spatial variability of $\bar{N}_t$ to increase when the primeval cosmic matter cools and eventually becomes non-relativistic. That is, one expects that gravitational perturbations from isotropy must necessarily increase with cosmic epoch in QMR. Thus no fine-tuning will be necessary to get a clumpy universe from a near-isotropic beginning.

A valid interpretation of equation (44) is that fixed operationally defined atomic units vary with epoch t in such a way that atomic length units shrink when t increases. This means that no matter can have been existing from the beginning of time since atomic length units increase without bound in the limit $t \rightarrow 0$. Consequently we may take an empty model described by equation (44) as an accurate cosmological model in this limit. Thus QMR yields a natural description of the beginning of time (with no physical singularity) where all big bang models fail (since big bang models are not valid for $t = 0$). But an empty beginning of the Universe means that one needs a working matter creation mechanism. Thus it is natural to suggest something analogous to particle creation by the expansion of the Universe in traditional big bang models. That is, in the very early Universe the global NKE is so strong that non-gravitational quantum fields cannot be treated as localized to sufficient accuracy, so one should get spontaneous pair production from excitations of vacuum fluctuations of such quantum fields (violating equation (25)). Moreover, newly created material particles should induce tiny gravitational perturbations which will grow when the Universe cools. The details of these suggestions have not been worked out. However, any hope that QMR may represent a complete framework for relativistic physics depends on if the mathematical details of a matter creation mechanism can be developed.

Even if models of the type (44) are not accurate for the present epoch we may still use it to illustrate some of the properties of a cosmological model where the expansion is non-kinematical. That is, the linear dependence of the scale factor on ct and the global positive curvature of space are valid predictions of any quasi-metric cosmological model, so even if a more realistic model with non-isotropic matter density does represent a deviation from (44), we may use (44) in combination with the equations of motion to deduce some general features of quasi-metric cosmology. In particular it is easy to derive the usual expansion redshift of momentum for decoupled massless particle species from

(44). To do that, use the coordinate expression for a null path in the χ-direction as calculated from (44) and the equations of motion. The result is [2]

$$\chi(t) = \chi(t_0) + \ln\frac{t}{t_0}, \tag{45}$$

and a standard calculation using (45) yields the usual expansion redshift formula. Also the corresponding time dilation follows from equation (45).

On the other hand the speed w of any inertial material point particle with respect to the FOs is unaffected by the global NKE [2]. In standard cosmology, however, the effect of cosmic expansion is that any inertial material particle will slow down over time with respect to the cosmic substratum. This difference illustrates that the nature of the global NKE is quite different from its kinematical counterpart in standard cosmology. But the constraints on the scale factor evolution coming from primordial nucleosynthesis should not depend critically on this difference though. That is, the fact that coasting universe models in metric gravity are consistent with primordial nucleosynthesis [8], indicates that this consistency holds for QMR as well.

Over the last few years a "concordance" big bang model has emerged from the observational determination of standard cosmological parameters. Key observational constraints on these cosmological parameters mainly come from two different types of data; i.e., from supernovae at cosmological distances (see the next section) and from analysis of temperature fluctuations in the cosmic microwave background (CMB). The concordance model predicts that the Universe should be nearly spatially flat and filled with exotic matter ("dark energy", possibly in the form of a cosmological constant) dominating the dynamics of the Universe, causing the cosmic expansion to accelerate. Since a universe dominated by dark energy raises some rather deep (and potentially unanswerable) questions concerning its nature, it could be argued that the inferred existence of the preposterous dark energy might be an artifact due to analyzing observational data within an incorrect space-time framework. To explore this possibility the CMB data should be reanalyzed within the quasi-metric framework. In particular this should be done for the data obtained from the Wilkinson Microwave Anisotropy Probe (WMAP). These data show at least one unexpected feature; namely that the temperature angular correlation function lacks power on angular scales greater than about 60° [9]. This may possibly have a natural explanation within the quasi-metric framework since any quasi-metric universe is closed and "small"; i.e., its spatial curvature scale should represent a natural cut-off for fluctuations. Moreover, since no counterpart to the Friedmann equation exists in QMR, there are no dynamical restrictions on the data ruling out a universe with scale factor close to the size of the observable Universe. This may be compared to the metric

framework, where a closed universe cannot be too small and have a trivial topology since this will be inconsistent with the value of the Hubble parameter obtained from other, independent observations (see, e.g., reference [10] for a further discussion of this point).

4.2 QMR and type Ia supernovae

Newtonian stars for which the equation of state takes the form $p \propto \rho_m^{\gamma}$, $\gamma > \frac{6}{5}$, are called *Newtonian polytropes* [11]. In quasi-metric theory it is possible to model Newtonian polytropes by taking Newtonian limits of the relevant equations but such that the t-dependence remains. According to quasi-metric theory the cosmic expansion is predicted to induce hydrostatic instabilities in polytropes consisting of degenerate matter [2]. But if the hydrodynamical effects on the gravitational field coming from instabilities can be neglected, one may solve the field equations for each epoch t assuming that the polytrope is at hydrostatic equilibrium. It is then possible to show [2] that the usual analysis of Newtonian polytropes [11] applies, but with with a variable G, i.e. $G \rightarrow G_t \equiv \frac{t}{t_0} G$.

Of particular interest are Newtonian polytropes for which $\gamma = \frac{4}{3}$, since such stars are models for Chandrasekhar mass white dwarfs (WDs). From reference [11] we easily find that the (passive) mass $m_{\rm c}$ and physical radius $\mathcal{L}_{\rm c}$ of such WDs (with identical central mass densities) depend on epoch such that $m_{\rm c}(t) = (\frac{t_0}{t})^{3/2} m_{\rm c}(t_0)$ and $\mathcal{L}_{\rm c}(t) = (\frac{t_0}{t})^{1/2} \mathcal{L}_{\rm c}(t_0)$, respectively. Since Chandrasekhar mass WDs are believed to be progenitors of type Ia supernovae, one may expect that any cosmic evolution of Chandrasekhar mass WDs should imply a systematic luminosity evolution of type Ia supernovae over cosmic time scales. However, such a luminosity evolution would be inconsistent with their use as standard candles when determining the cosmological parameters in standard cosmology: luminosity evolution could have serious consequences for an interpretation of the supernova data in terms of an accelerating cosmic expansion indicating a non-zero cosmological constant [12], [13].

Now quasi-metric theory predicts that the cosmic expansion does neither accelerate nor decelerate. Moreover, according to quasi-metric theory, the Chandrasekhar mass varies with epoch and this means that type Ia supernovae may be generated from cosmologically induced collapse of progenitor WDs. But the consequences for type Ia supernova peak luminosities due to the predicted evolution of progenitor WDs are not clear. Since the luminosity of type Ia supernovae comes from γ-disintegration of unstable nuclear species (mainly ^{56}Ni) synthesized in the explosion, this luminosity could depend critically on the conditions of the nuclear burn. That is, the detailed nuclear composition synthesized in the explosion might depend on the local acceleration of gravity experienced

by the burning front (which should depend on the value of G). Other critical factors might be supernova progenitor mass and chemical composition [14] (see below). Also the presence of more massive ejecta during the explosion could have an influence on supernova luminosities and light curves.

The effects of a varying gravitational "constant" on type Ia supernova luminosities have been studied elsewhere [15], [16]. In these papers it is assumed that the peak luminosity is proportional to the synthesized mass of ^{56}Ni which again is assumed to be proportional to the Chandrasekhar mass. Moreover, using a toy model of the supernova explosion, the dependence of the intrinsic time scale $\tau_{\rm s}$ of the explosion on G has been deduced (neglecting radioactive heating). The results are

$$L(t)\propto\Big(\frac{G(t)}{G(t_0)}\Big)^{-\frac{3}{2}}, \qquad \tau_{\rm s}(t)\propto\Big(\frac{G(t)}{G(t_0)}\Big)^{-\frac{3}{4}}. \tag{46}$$

We see that if both of these luminosity and intrinsic time scale evolutions were valid within the quasi-metric framework, we would have been forced to deduce that $L(z)\propto(1+z)^{\frac{3}{2}}$ and $\tau_{\rm s}(z)\propto(1+z)^{\frac{3}{4}}$. This means that, rather than intrinsically fainter, ancient supernovae would have been predicted to be intrinsically *brighter* than today's in addition to displaying broader light curves, contrary to observations. But it is by no means obvious that the toy model yielding the above luminosity and intrinsic time scale evolutions is sufficiently realistic. Rather one should include all aspects of progenitor evolution on type Ia supernovae before deducing such evolutions. In particular the amount of ^{56}Ni synthesized in the explosion and thus the supernova luminosity will depend on the C/O ratio present in the progenitor WD. In general more massive progenitor WDs will have smaller C/O ratios than less massive ones due to different conditions in the He-burning stages of their progenitor stars [17]. A smaller C/O ratio implies a that a smaller percentage of ^{56}Ni should be produced relative to other explosion products [17]. Besides, a smaller C/O ratio will affect the energetics of the explosion such that ejecta velocities and thus supernova size will be smaller at any given time after the explosion [17]. Since these effects may be important for supernova luminosity evolution, it is possible that ancient supernovae in fact could be intrinsically dimmer than today's, despite their being more massive. But any investigation of this possibility requires detailed numerical simulations, so before such have been performed it is not possible to say whether or not the predictions from quasi-metric theory are consistent with the data.

However, what we *can* easily do is to see if it is possible to construct a simple luminosity evolution which, in combination with the cosmological toy model (44), yields a reasonable fit to the supernova data. That is, we may try a luminosity evolution of the

source of the form

$$L_{\rm qmr} = L_{\rm std}(\frac{t_0}{t})^{\epsilon} = L_{\rm std}(1+z)^{-\epsilon}, \tag{47}$$

where $L_{\rm std}$ is a fixed standard luminosity, and see if the data are well fitted for some value(s) of ϵ. To check this we use the postulated luminosity evolution to plot apparent magnitude $m_{\rm qmr}$ versus redshift for type Ia supernovae. The easiest way to compare this to data is to calculate the predicted difference between the quasi-metric model (with source luminosity evolution) and a FRW model where the scale factor increases linearly with epoch, namely the "expanding Minkowski universe" given by a piece of Minkowski space-time:

$$ds^2 = -(dx^0)^2 + (x^0)^2\Big(d\chi^2 + \sinh^2\chi d\Omega^2\Big). \tag{48}$$

The difference in apparent magnitude Δm between the two models (as a function of redshift z) can be found by a standard calculation. The result is

$$\begin{aligned} \Delta m \equiv m_{\rm qmr} - m_{\rm min} \quad = \quad & 2.5\log_{10}\Big[\sin^2\{\ln(1+z)\}\Big] \\ & -5\log_{10}\Big[\sinh\{\ln(1+z)\}\Big] - 2.5\log_{10}\frac{L_{\rm qmr}}{L_{\rm min}}, \end{aligned} \tag{49}$$

where $L_{\rm qmr}/L_{\rm min}$ represents the luminosity evolution of the source in our quasi-metric model relative to no luminosity evolution in an empty FRW model. One may then find the relation $m_{\rm qmr}(z)$ from equation (49) and the relation $m_{\rm min}(z)$ graphically shown in reference [12], and then compare to data. One finds that the quasi-metric model is quite consistent with the data for values of ϵ of about 0.5. That is, for ϵ near 0.5 Δm has a maximum at $z{\approx}0.5$; for higher redshifts Δm decreases (and eventually becomes negative for z larger than about 1.2). In standard cosmology this behaviour would be interpreted as evidence for an era of cosmic deceleration at high z.

We conclude that quasi-metric cosmology combined with a simple luminosity evolution seems to be consistent with the data but that a much more detailed model should be constructed to see if the postulated luminosity evolution has some basis in the physics of type Ia supernovae.

From the above we see that the assertion that type Ia supernovae can be used as standard cosmic candles independent of cosmic evolution is a model-dependent assumption. But the fact is that models for which this holds fail to explain the effects of the cosmic expansion seen in the solar system [5], [6]. Thus, any interpretation of the supernova data indicating that the expansion of the Universe is accelerating should be met with some extra skepticism.

5 Conclusions

In many ways any theory of gravity compatible with the quasi-metric framework must be fundamentally different from metric theories of gravity since quasi-metric space-time is not modeled as a semi-Riemannian manifold. The most obvious of these differences is the existence of a non-metric sector and the fact that it directly influences the equations of motion. This means that the existence of a non-metric sector may be inferred from data on test particle motion. In fact non-metric effects on test particle motion in weak gravitational fields can be tested against experiment rather independently of any systematic weak field expansion for the metric sector. And the status so far is that it seems like non-metric effects are seen in good agreement with predictions [5], [6].

The geometrical structure of quasi-metric space-time also gives QMR some conceptual advantages over metric theory. For example, the simple causal structure of QMR leaves no room for event horizons. Thus potential vexing questions concerning the nature of black holes and space-time singularities do not exist in QMR. Besides, quasi-metric space-time is constructed to yield maximal predictive power. This shows up most clearly in cosmology, since both the global shape and curvature of the Universe in addition to its expansion history are basic features of QMR and not adjustable. This is in contrast to metric theory where essentially arbitrary cosmic initial conditions and a number of cosmological parameters are available, making it much more flexible than QMR. Should QMR turn out to survive confrontation with cosmological observations, the vulnerability of QMR predictions would give strong support to QMR.

But even in its metric sector quasi-metric gravity is different from those metric theories suitable for a standard PPN-analysis. The main reason for this is that quasi-metric gravity contains an *implicit* dynamical degree of freedom not coupled explicitly to matter. Unfortunately the lack of a weak field expansion formalism (having some necessary correspondence with the PPN-framework) makes it harder to test the metric aspects of QMR. However, a weak field expansion scheme is not needed to see that the geometric structure of quasi-metric space-time is consistent with no Nordtvedt effect. In other words, the quasi-metric theory of gravity presented in this paper should fulfill the Gravitational Weak Equivalence Principle (GWEP). In fact many experiments testing the validity of the SEP actually test the GWEP. Thus the fact that QMR violates the SEP needs not be fatal.

But there are other crucial observational tests which QMR has to survive. For example, the nature of gravitational radiation in QMR should be worked out to see if predictions are compatible with observations of binary pulsars. So there is much further

work to be done before we can know whether or not QMR is viable. However, observations do seem to confirm the existence of a non-metric sector. This suggests that metric theory is wrong so QMR sails up as a potential alternative.

Acknowledgment

I wish to thank Dr. Kåre Olaussen for making a critical review of the manuscript.

References

[**1**] C.M. Will, *Theory and Experiment in Gravitational Physics*, Cambridge University Press (1993).

[**2**] D. Østvang, *Doctoral Thesis*, (2001) (gr-qc/0111110).

[**3**] M. Castagnino, L. Lara, O. Lombardi, *Class. Quantum Grav.* **20**, 369 (2003) (quant-ph/0211162).

[**4**] D. Østvang, gr-qc/0303107.

[**5**] D. Østvang, gr-qc/0201097.

[**6**] D. Østvang, *Class. Quantum Grav.* **19**, 4131 (2002) (gr-qc/9910054).

[**7**] J.D. Anderson, P.A. Laing, E.L. Lau, A.S Liu, M.M Nieto, S.G. Turyshev, *Phys. Rev. Lett.* **81**, 2858 (1998) (gr-qc/9808081).

[**8**] M. Sethi, A. Batra, D. Lohiya, *Phys. Rev.* D **60**, 108301 (1999) (astro-ph/9903084).

[**9**] D.N. Spergel *et al.*, *Astrophys. J. Suppl.* **148**, 175 (2003) (astro-ph/0302209).

[**10**] G. Efstathiou, *Mont. Not. R. Astron. Soc.* **343**, L95 (2003) (astro-ph/0303127).

[**11**] S. Weinberg, *Gravitation and Cosmology*, John Wiley & Sons, Inc. (1972).

[**12**] S. Perlmutter *et al.*, *ApJ* **517**, 565 (1999) (astro-ph/9812133).

[**13**] A.G. Riess *et al.*, *AJ* **116**, 1009 (1998) (astro-ph/9805201).

[**14**] P. Höflich, K. Nomoto, H. Umeda, J.C. Wheeler, *ApJ* **528**, 590 (2000) (astro-ph/9908226).

[**15**] A. Riazuelo, J.-P. Uzan, *Phys. Rev.* D **66**, 023525 (2002).

[**16**] E. Gastañaga, E. García-Berro, J. Isern, E. Bravo, I. Domínguez, *Phys. Rev.* D **65**, 023506 (2002).

[**17**] I. Domínguez, P. Höflich, O. Stranerio, M. Limongi, A. Chieffi, in *Proceedings of IAU Colloquium 192* (2003), Springer Verlag (astro-ph/0311140).